



NEXT LEVEL QUANTUM INFORMATION PROCESSING FOR SCIENCE AND TECHNOLOGY

DELIVERABLE D1.1 – REPORT ON SCALABLE QUDIT CHARACTERIZATION TECHNIQUES, ERROR MODEL, AND CERTIFICATION OF THE QUANTUM PERFORMANCE

Grant Agreement:	101080086
Project Title:	NEXT LEVEL QUANTUM INFORMATION PROCESSING FOR SCIENCE AND TECHNOLOGY
Project Acronym:	NeQST
Project Start Date:	01 November 2022
Deliverable Number and title:	D1.1 – Report on scalable qudit characterization techniques, error model, and certification of the quantum performance
Related Work Package:	WP1 – Advanced qudit control and design
Due Date:	30 April 2024
Submission date:	30 April 2024
Type:	R
Dissemination Level:	Public



Funded by
the European Union

Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the granting authority can be held responsible for them.

Authors

	Name	Beneficiary
Lead Editor	Martin Ringbauer	UIBK
Contributors	Antonio Acin	ICFO
	Robert Wille	TUM
	Philipp Hauke	UNITN
	Sara Rebecchi	UNITN

History of change

Version	Date	Contributor	Additional information
0.1	07.02.2024	Martin Ringbauer (UIBK)	Rough draft of content
1.0	14.03.2024	Martin Ringbauer (UIBK)	First draft finished
1.1	27.03.2024	Antonio Acín (ICFO)	Revision of draft
1.2	09.04.2024	Philipp Hauke (UNITN)	Revision of draft
2.0	22.04.2024	Martin Ringbauer (UIBK)	Final draft
2.0	24.04.2024	Philipp Hauke, Sara Rebecchi (UNITN)	Quality check and finalisation of the document

Executive Summary

This report outlines the developments within NeQST towards scalable qudit characterization techniques, error model, and certification of the quantum performance. Compared to their simpler two-level counterpart, qudits present a range of new challenges due to the richer Hilbert space structure.

Characterization techniques must be carefully designed to avoid unnecessary overheads. Here, scalable techniques were developed that exploit inherent correlations between calibration parameters and use continuous compensation rather than tedious characterization. This enables the compensation of unwanted phase shifts, a primary challenge in qudits, with minimal calibration overhead.

Noise models become more involved than for qubits, where phenomenological depolarizing noise goes a long way. In trapped ion systems, however, noise is generally driven by gate operations. As a consequence, noise in qudit systems, where gates generally act on subspaces, is not well described by depolarizing noise. While microscopic noise modelling accurately describes the qudit gates, this is impractical for circuit simulations pursued within NeQST. Hence, we developed a phenomenological noise model that is easy to handle in classical simulations and closely approximates the system performance.

Finally, the development of software phase gates enables software phase tracking for entangling gates, as well as optimized pulse sequences. The latter is demonstrated to reduce the gate count and thereby increase the fidelity of local operations by over a factor of 2.

Table of Contents

EXECUTIVE SUMMARY	3
TABLE OF CONTENTS	4
1. INTRODUCTION.....	5
1.1 RELATIONSHIP TO PROJECT WORK PROGRAM	5
1.2 RELEVANCE OF THE PROBLEM	5
1.3 STATE OF THE ART	5
2. PROGRESS ACHIEVED THROUGH NEQST	5
2.1 QUDIT CHARACTERIZATION AND CALIBRATION	5
2.2 QUDIT NOISE MODEL	7
2.3 CERTIFICATION OF THE QUANTUM PERFORMANCE	8
3. CONCLUSIONS.....	9
4. REFERENCES	9

1. Introduction

This Deliverable of the project NeQST constitutes a report on our activities on the characterization of trapped-ion qudit hardware, applicable noise models, and benchmarking within work package (WP) 1 of the project. In this introduction, we relate the Deliverable to the project work program, discuss the importance of the problem, and present the state of the art in qudit quantum hardware characterization and control methods.

1.1 Relationship to project work program

This deliverable is a central element of Work Package (WP) 1, *Advanced Qudit Control and Design*. Within this WP, we aim to develop scalable characterization tools for qudit systems, benchmarks for various performance aspects, as well as automated design and simulation tools to make efficient use of the new resources. The work is carried out in close collaboration between the University of Innsbruck (UIBK) and the Technical University of Munich (TUM), as well as The Institute of Photonic Sciences (ICFO), with vital contributions and input from all project partners: UNITN, IOSB-AST, HRI.

1.2 Relevance of the problem

Qudit quantum computing hardware is in the very early stages of development and lacks much of the quantum information framework and standardized toolboxes enjoyed by qubit-based systems. Particularly, the characterization of high-dimensional Hilbert spaces is a major obstacle that must be addressed to make qudit-systems competitive in a rapidly developing quantum computing environment. Since not only fundamental Hilbert space structures but also the types of logic gates are very different in qudits, most of the results from binary systems are not applicable or must be carefully generalized. One critical aspect in this vein is understanding and modeling of noise processes, which is central for assessing potential benefit of using qudit hardware for a problem of interest.

1.3 State of the art

Automated characterization methods are well established for qubit-based trapped-ion systems, covering most aspects of the device with minimal impact on the duty cycle. Yet most of these methods are not directly applicable to higher-dimensional systems, or come with a significant overhead that affects the device's duty cycle. Prior to the project, UIBK has already demonstrated randomized benchmarking (RB) for local qudit gates [MR2022]. However, contrary to the qubit case, RB methods are not applicable to most (non-Clifford) qudit entangling gates, such that alternative approaches are needed. Similarly, effective dephasing and depolarizing noise models commonly used for modeling the noise in qubit devices are not applicable for qudit systems, which are affected by a much wider range of noise processes.

2. Progress achieved through NeQST

2.1 Qudit characterization and calibration

Naively, there are of the order of d^2 (or more) parameters to be calibrated for operations in a d -dimensional Hilbert space. However, these parameters are typically not independent, but often highly correlated through the underlying atomic physics model. One example is the set of transition frequencies between the various qudit levels, which depend on the atomic structure and the applied magnetic field, and various additional corrections such as quadrupole shifts. Notably, the only variable quantity for the transition frequencies is the magnetic field, such that there is no overhead in calibrating the transition frequencies of d rather than 2 levels in the experiment.

For quantum gate operations, one additionally has to consider the atom-light interaction, which depends on the light polarization and amplitude. Through NeQST, we developed calibration routines that focus on maintaining the single parameter laser power, rather than calibrating each individual Rabi frequency, to achieve the calibration of the full local qudit gate set with no overhead compared to qubits. The key challenge then becomes the characterization and calibration of entangling gate operations. Here, we focus on Molmer-Sorenson, and Cirac-Zoller-type gates. In both cases, the gate action is restricted to a subspace of the qudit Hilbert space, which means that the calibration of the gates themselves incurs no overhead compared to qubits. However, in qudit systems, one has to consider AC Stark shifts experienced by spectator levels during quantum gate operations. Since this effect is not present for qubits, UIBK developed new methods for addressing it. On the one hand, these shifts can be removed by adding additional off-resonant laser fields that physically balance the Stark shifts on the various transitions [MM2023]. On the other hand, building on the developed tools for “software” phase shifts, these effects are compensated by tracking the various phase shifts (c.f. M1, M2). This leads to a calibration overhead that is linear in the qudit dimension.

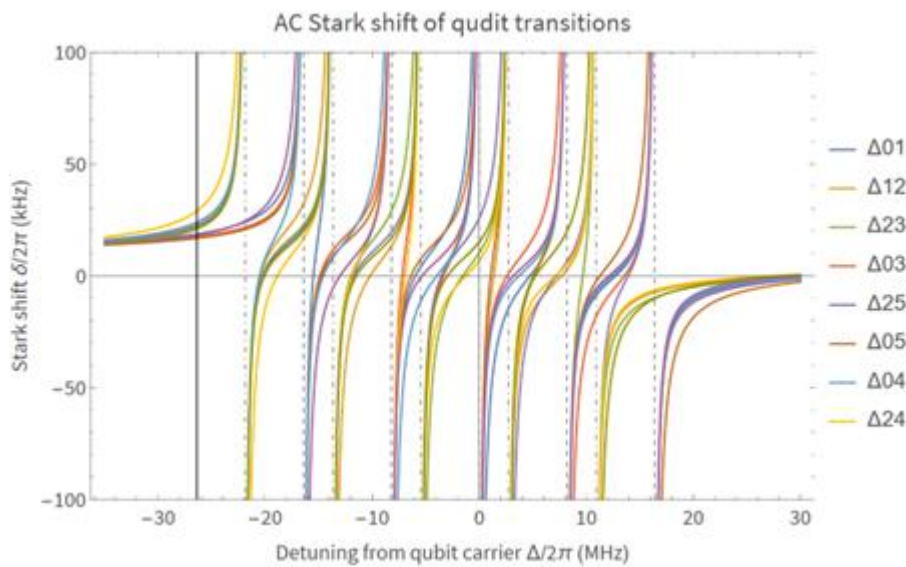


Figure 1: AC Stark shifts of various qudit transitions due to an external laser field

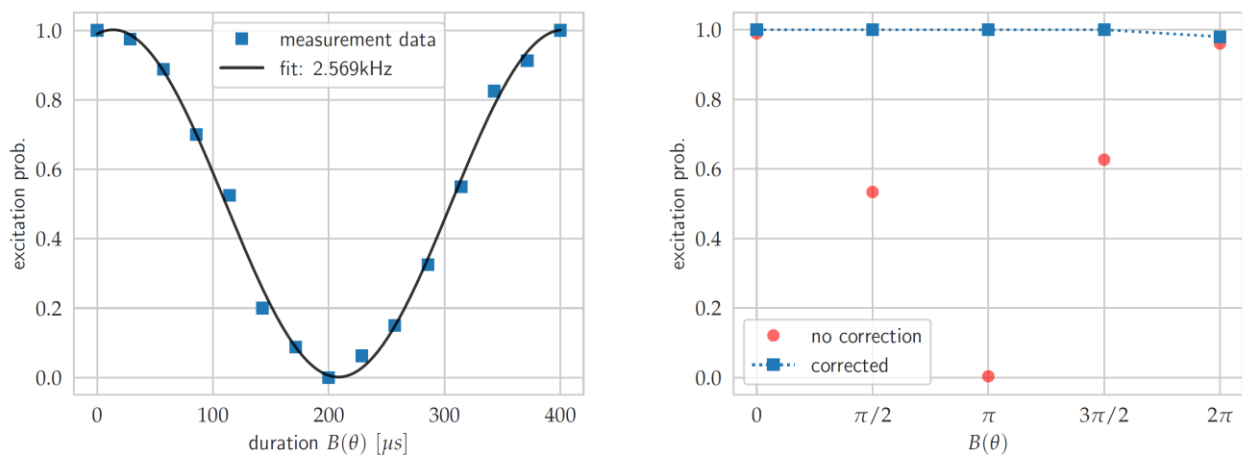


Figure 2: Continuous compensation of AC Stark shifts during an entangling gate through the use of an additional compensation field. (Left) Shift calibration and fit (Right) Gate performance with and without continuous compensation.

2.2 Qudit noise model

Noise models for quantum processors span a wide range, from microscopic modelling of light-matter interaction, via effective noise models at the gate level, to effective models for the performance of deep quantum circuits. In trapped-ion qubit devices, these regimes have been studied extensively. Effective noise models for such devices are routinely used to gain a rough understanding of circuit performance through simulators such as the widely established qiskit as well as the solutions developed within NeQST. Detailed microscopic models, on the other hand, are used to identify bottlenecks in gate performance. In qudit systems, the situation, however, is quite different. Not only does the high-dimensional Hilbert space complicate the numerical simulations necessary for developing microscopic noise models, but also effective models based on (generalized) Pauli noise channels fail to characterize the systems accurately.

In the context of newly developed qudit controlled-rotation (crot) gates [MM2023], we systematically studied the effects of gate-level noise and the suitability of effective noise models (also related to M3). The leading noise sources in trapped-ion qudits tend to be related to laser phase and amplitude noise, motional mode heating or decoherence, uncontrolled Stark shifts, and magnetic field fluctuations [PH2023]. Only the latter affects idle qudits and also acts differently for each qudit level. The other noise sources are related to the gate operations and primarily affect entangling gates due to their higher laser power requirement and increased sensitivity. With this microscopic understanding, it is clear that generic noise models such as generalized dephasing or depolarizing noise, are unlikely to capture the behaviour of the system, which we verified in the context of [MM2023].

In order to model the noise more accurately, a physics-inspired noise model is considered for the used crot entangling gates, while local operations are assumed to be approximately noiseless. For the used gate mechanism, laser amplitude and phase fluctuations lead to effective depolarizing noise in the subspace that is addressed by the gate. At the same time, the gate light field leads to systematic AC Stark shifts of the spectator levels. While the latter are compensated on average through the above discussed phase tracking, amplitude fluctuations of the gate laser field also lead to fluctuations in the AC Stark shifts. This effect is thus described as effective dephasing noise on the subspace where the gate *does not* act. In principle, the strength of the dephasing varies for different qudit levels and there is an additional state-dependent dephasing from magnetic field fluctuations. Hence, while it can sometimes be reasonable as a first approximation to treat the dephasing strength as equal, for example, when applying an effective channel at the end of the circuit, a reliable noise model would need to take the unequal strength into account. The resulting noise model can be integrated into the simulation framework developed by TUM, and will inform the developments in WP2 and WP3.

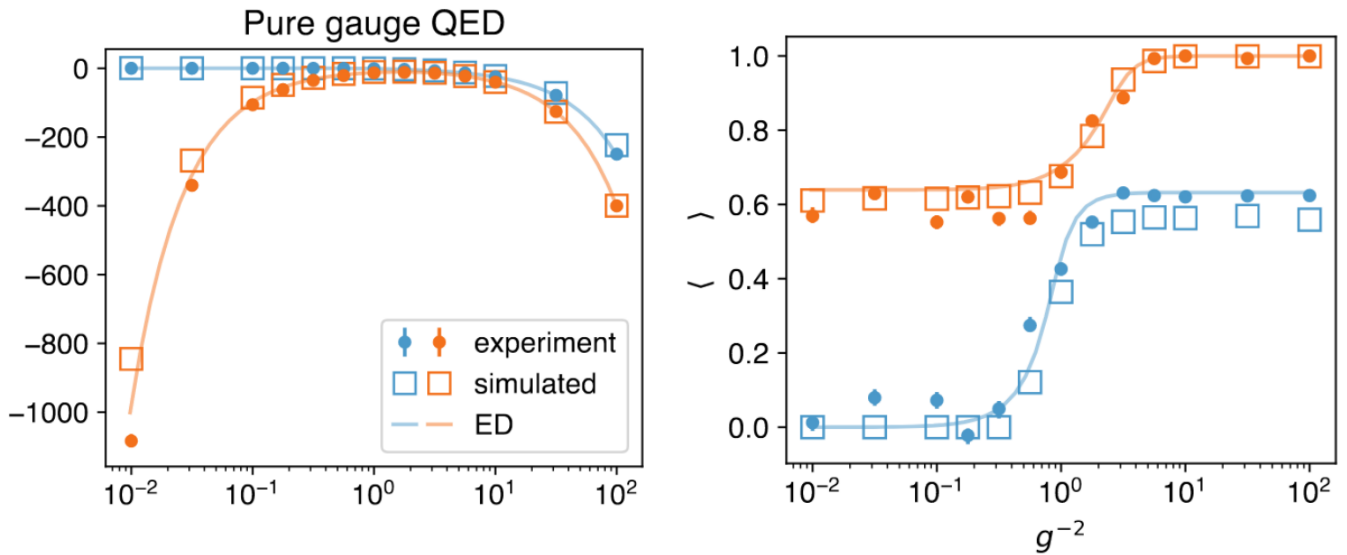


Figure 3: Simplified noise model consisting of depolarizing noise on the active transition and dephasing noise on spectator levels approximates the circuit performance well [MM2023].

2.3 Certification of the quantum performance

The standard approach to quantifying the performance of quantum gate operations has become randomized benchmarking (RB). RB is based on the use of a random sequence of Clifford gates, which is classically efficiently inverted to measure the probability of ending in the starting state. The decay of this so-called survival probability with the number of gates can then be related to an average gate error rate. UIBK had already demonstrated the use of RB for prime-dimensional qudits prior to NeQST [MR2022]. Through the project, UIBK and TUM developed efficient and adaptive compilation methods, which, combined with software-based tracking of the phase evolution of all qudit levels, developed within the project, enables an improvement in quantum gate performance by a factor of 2. This is reflected in the RB measurements.

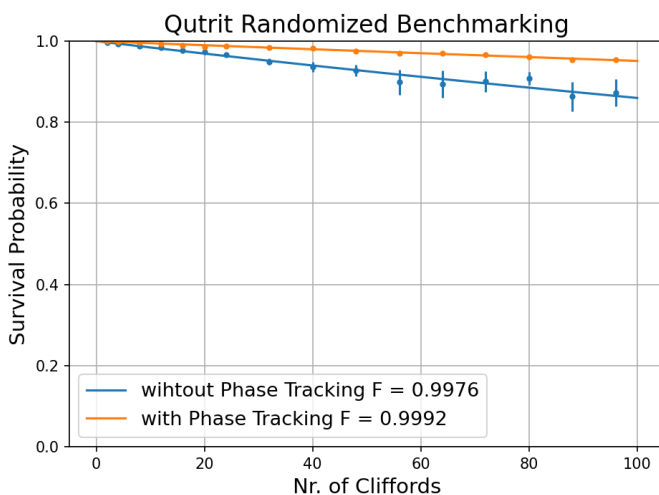


Figure 4: Randomized benchmarking results show an improvement in average qutrit Clifford gate fidelity by over a factor of 2 through the use of software phase tracking.

The typically more challenging aspect of characterizing quantum processors is entangling operations. In the case of qubits, standard gates such as the CNOT are part of the two-qubit Clifford group and can thus easily be characterized using generalized RB-type methods. For qudits, however, the situation is significantly more complicated, since the native qudit gates are non-Clifford

operations. While developing benchmarking routines for non-Clifford gates is beyond the scope of this project, efforts are under way at UIBK. Notably, however, these operations are still cyclic, which makes it possible to characterize them by examining the return probability after a sequence of identical gates. At UIBK, suitable procedures have been developed for Molmer-Sorenson [MR2022], light-shift [PH2023] and Cirac-Zoller-type [MM2023] qudit gates.

3. Conclusions

In the context of Deliverable D1.1, the consortium developed the planned new techniques for characterizing qudit quantum processors efficiently; gained valuable insights into the applicable noise models at the microscopic, as well as at the simplified effective level; implemented and demonstrated virtual phase gates that result in a factor of 2 improvement in local qudit gate performance. These results significantly improve the flexibility and reliability of qudit-based quantum information processing. Beyond the current capabilities, the developed methods are a stepping stone, allowing us to explore more involved qudit entangling gate operations, which have the potential to greatly reduce the required gate count for quantum simulation applications [GC2024]. The resulting toolbox would significantly extend the reach of qudit quantum processors and provide increased design flexibility for applications such as lattice gauge theory simulations pursued within NeQST.

The results of this Deliverable also provide the basis for the corresponding automatic methods for qudit design and qudit simulation. An initial (alpha) version of this is already publicly available at <https://github.com/cda-tum/mqt-qudits/> and will be further extended within this project. Besides that, this Deliverable will inform the further progress in the project. For examples, the resulting enhanced performance of the trapped-ion qudit processor will enable improved quantum simulations, such as are performed within WP2 of NeQST, and more coherent quantum-optimization protocols, as are explored for industry-relevant application problems within WP3.

4. References

- [MR2022] Ringbauer, Meth, Postler, Stricker, Blatt, Schindler, Monz, A universal qudit quantum processor with trapped ions, *Nature Physics* 18, 1053 (2022)
- [PH2023] Hrmo, Wilhelm, Gerster, van Mourik, Huber, Blatt, Schindler, Monz, Ringbauer, Native qudit entanglement in a trapped ion quantum processor, *Nature Communications* 14, 2242 (2023)
- [MM2023] Meth, Haase, Zhang, Edmunds, Postler, Steiner, Jena, Dellantonio, Blatt, Zoller, Monz, Schindler, Muschik, Ringbauer, Simulating 2D lattice gauge theories on a qudit quantum computer, [arXiv:2310.12110](https://arxiv.org/abs/2310.12110),
- [GC2024] Calajò, Magnifico, Edmunds, Ringbauer, Montangero, Silvi, Digital quantum simulation of a (1+1)D SU(2) lattice gauge theory with ion qudits, [arxiv:2402.07987](https://arxiv.org/abs/2402.07987) (2024)